

ELECTRICAL CONDUCTION IN THE EARLY UNIVERSE

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The electrical conductivity has been calculated¹ in the early universe at temperatures below as well as above the electroweak vacuum scale, $T_c \simeq 100\text{GeV}$. Debye and dynamical screening of electric and magnetic interactions leads to a finite conductivity, $\sigma_{el} \sim T/\alpha \ln(1/\alpha)$, at temperatures well below T_c . At temperatures above, W^\pm charge-exchange processes – analogous to color exchange through gluons in QCD – effectively stop left-handed charged leptons. However, right-handed leptons can carry current, resulting in σ_{el}/T being only a factor $\sim \cos^4 \theta_W$ smaller than at temperatures below T_c .

1 Introduction

The strong magnetic fields measured in many spiral galaxies, $B \sim 2 \times 10^{-6}$ G, are conjectured to be produced primordially; proposed mechanisms include fluctuations during an inflationary universe² or at the GUT scale, and plasma turbulence during the electroweak transition or in the quark-gluon hadronization transition. The production and later diffusion of magnetic fields depends crucially on the electrical conductivity, σ_{el} , of the matter in the universe; typically, over the age of the universe, t , fields on length scales smaller than $L \sim (t/4\pi\sigma_{el})^{1/2}$ are damped.

The electrical conductivity was estimated in² in the relaxation time approximation as $\sigma_{el} \sim n\alpha\tau_{el}/m$ with $m \sim T$ and relaxation time $\tau_{el} \sim 1/(\alpha^2 T)$, where $\alpha = e^2/4\pi$. In Refs.^{3,4} the relaxation time was corrected with the Coulomb logarithm. A deeper understanding of the screening properties in QED and QCD plasmas has made it possible to calculate a number of transport coefficients including viscosities, diffusion coefficients, momentum stopping times, etc., exactly in the weak coupling limit^{5,6,7,8}. However, calculation of processes that are sensitive to very singular forward scatterings remain problematic. For example, the calculated color diffusion and conductivity⁹, even with dynamical screening included, remain infrared divergent due to color exchange in forward scatterings. Also the quark and gluon damping rates at non-zero momenta calculated by resumming ring diagrams exhibit infrared divergences¹⁰ whose resolution requires more careful analysis including higher order diagrams as, e.g., in the Bloch-Nordsieck calculation of Ref.¹¹. Charge exchanges through W^\pm , processes similar to gluon color exchange in QCD, are important in forward scatterings at temperatures above the W mass, M_W .

2 Electrical conductivities in high temperature QED

The electrical conductivity in the electroweak symmetry-broken phase at temperatures below the electroweak boson mass scale, $M_W \gg T \gg m_e$, is dominated by charged leptons $\ell = e^-, \mu^-, \tau^-$ and anti-leptons $\bar{\ell} = e^+, \mu^+, \tau^+$ currents. In the broken-symmetry phase weak interactions between charged particles, which are generally smaller by a factor $\sim (T/M_W)^4$ compared with photon-exchange processes, can be ignored. The primary effect of strong interactions is to limit the drift of strongly interacting particles, and we need consider only electromagnetic interactions between charged leptons and quarks.

Transport processes are most simply described by the Boltzmann kinetic equation for the distribution functions of particle species i , of charge e . We use the standard methods of linearizing around equilibrium with a collision term with interactions given by perturbative QED. We refer to ¹ for details of the calculation but note the essential physics of Debye screening of the longitudinal (electric) interactions and Landau damping of the transverse (magnetic) interactions. The electrical conductivity for charged leptons is to leading logarithmic order ^{6,1}

$$\sigma_{el}^{(\ell\bar{\ell})} \equiv j_{\ell\bar{\ell}}/E = \frac{3\zeta(3)}{\ln 2} \frac{T}{\alpha \ln(C/\alpha N_l)}, \quad m_e \ll T \ll T_{QGP}. \quad (1)$$

where the constant $C \sim 1$ in the logarithm gives the next to leading order terms^{12,7}. Note that the number of lepton species drops out except in the logarithm. The above calculation taking only electrons as massless leptons ($N_l = 1$) gives a first approximation to the electrical conductivity in the temperature range $m_e \ll T \ll T_{QGP}$, below the hadronization transition, $T_{QGP} \sim 150$ GeV, at which hadronic matter undergoes a transition to a quark-gluon plasma. Thermal pions and muons in fact also reduce the conductivity by scattering electrons, but they do not become significant current carriers because their masses are close to T_{QGP} .

For temperatures $T > T_{QGP}$, the matter consists of leptons and deconfined quarks. The quarks themselves contribute very little to the current, since strong interactions limit their drift velocity. However, they are effective scatterers, and thus modify the conductivity (see ^{6,1} and Fig. 1):

$$\sigma_{el} = \frac{N_l}{N_l + 3 \sum_q^{N_q} Q_q^2} \sigma_{el}^{(\ell\bar{\ell})}, \quad T_{QGP} \ll T \ll M_W. \quad (2)$$

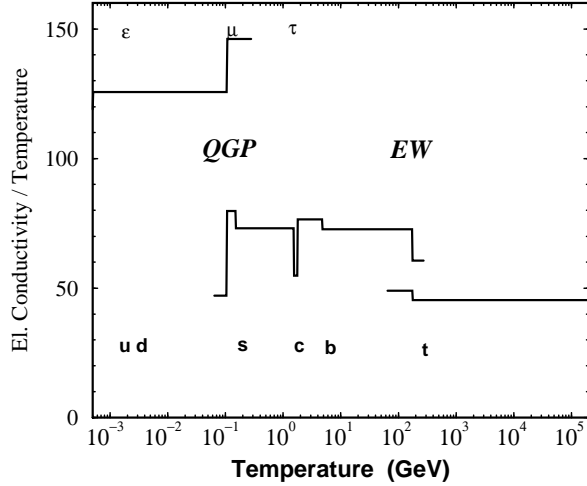


Figure 1: The ratio of the electrical conductivity to temperature, σ_{el}/T vs. temperature. The temperatures, where the transitions from hadronic to quark-gluon plasma and electroweak symmetry breaking occur, are indicated by QGP and EW respectively. The conductivity σ_{el} is given by Eqs. (1,2,4) in the three regions and are extrapolated into the regions of the phase transitions. The quark and lepton masses in the figure indicate the temperatures at which they are thermally produced and thus affect the conductivity. For clarity, the quarks and leptons are assumed to make their appearance abruptly when T becomes $> m_i$; in reality they are gradually produced thermally as T approaches their masses.

3 The symmetry-restored phase

To calculate the conductivity well above the electroweak transition, $T \gg T_c \simeq 100\text{GeV}$ ¹³, where the electroweak symmetries are fully restored, we describe the electroweak interactions by the standard model Weinberg-Salam Lagrangian with minimal Higgs (ϕ). At temperatures below T_c , the Higgs mechanism naturally selects the representation W^\pm , Z^0 , and γ of the four intermediate vector bosons. At temperatures above the transition – where $\langle\phi\rangle$ vanishes for a sharp transition, or tends to zero for a crossover – we consider driving the system with external vector potentials $A^a = B, W^\pm, W^3$, which give rise to corresponding “electric” fields $E_i^a \equiv F_{i0}^a = \partial_i A_0^a - \partial_0 A_i^a$ for $A^a = B$ and $F_{i0}^a = \partial_i A_0^a - \partial_0 A_i^a - g\epsilon_{abc}A_i^b A_0^c$ for $A^a = W^1, W^2, W^3$. One can equivalently drive the system with the electromagnetic and weak fields derived

from A , Z^0 , and W^\pm , as when $T \ll T_c$, or any other rotated combination of these. We consider here only the weak field limit and ignore the nonlinear driving terms. The self-couplings between gauge bosons are important, however, in the scattering processes in the plasma determining the conductivity.

The electroweak fields A^b act on the matter to generate currents J_a of the various particles present in the plasma, such as left and right-handed leptons and their antiparticles, and quarks, vector bosons, and Higgs bosons. The Higgs and vector boson contributions are negligible. Therefore the significant terms in the currents are $J_B^\mu = \frac{g'}{2}(\bar{L}\gamma^\mu Y L + \bar{R}\gamma^\mu Y R)$ and $J_{W^i}^\mu = \frac{g}{2}\bar{L}\gamma^\mu \tau_i L$. We define the conductivity tensor σ_{ab} in general by

$$\mathbf{J}_a = \sigma_{ab} \mathbf{E}^b. \quad (3)$$

The electroweak $U(1) \times SU(2)$ symmetry implies that the conductivity tensor, σ_{ab} , in the high temperature phase is diagonal in the representation, as can be seen directly from the (weak field) Kubo formula which relates the conductivity to (one-boson irreducible) current-current correlation functions. The construction of the conductivity in terms of the Kubo formula assures that the conductivity and hence the related entropy production in electrical conduction are positive. Then $\sigma = \text{Diag}(\sigma_{BB}, \sigma_{WW}, \sigma_{WW}, \sigma_{WW})$. Due to isospin symmetry of the W -interactions the conductivities $\sigma_{W^i W^i}$ are the same, $\equiv \sigma_{WW}$, but differ from the B -field conductivity, σ_{BB} .

The calculation of the conductivities σ_{BB} and σ_{WW} in the weak field limit parallels that done for $T \ll T_c$. The main difference is that weak interactions are no longer suppressed by a factor $(T/M_W)^4$ and the exchange of electroweak vector bosons must be included. The conductivity, σ_{BB} , for the abelian gauge field B can be calculated similarly to the electrical conductivity at $T \ll T_c$.

Although the quarks and W^\pm are charged, their drifts in the presence of an electric field do not significantly contribute to the electrical conductivity. Charge flow of the quarks is stopped by strong interactions, while similarly flows of the W^\pm are effectively stopped by $W^+ + W^- \rightarrow Z^0$, via the triple boson coupling. Charged Higgs bosons are likewise stopped via $W^\pm \phi^\dagger \phi$ couplings. These particles do, however, affect the conductivity by scattering leptons. Particle masses have negligible effect on the conductivities¹.

These considerations imply that the B current consists primarily of right-handed e^\pm , μ^\pm and τ^\pm , interacting only through exchange of uncharged vector bosons B , or equivalently γ and Z^0 . Because the left-handed leptons interact through \mathbf{W} as well as through B , they give only a minor contribution to the current. They are, however, effective scatterers of right-handed leptons. The

resulting conductivity is (see ¹ for details)

$$\sigma_{BB} = \frac{9}{19} \cos^2 \theta_W \sigma_{el}, \quad T \gg T_c. \quad (4)$$

Applying a W^3 field to the electroweak plasma drives the charged leptons and neutrinos oppositely since they couple through $g\tau_3 W_3$. In this case, exchanges of W^\pm dominate the interactions as charge is transferred in the singular forward scatterings and Landau damping is not sufficient to screen the interaction, a QCD magnetic (gluon) mass, $m_{mag} \sim g^2 T$, will provide an infrared cutoff. We expect that $\sigma_{WW} \sim \alpha \sigma_{BB}$. This effect of W^\pm exchange is analogous to the way gluon exchange in QCD gives strong stopping and reduces the “color conductivity” significantly ⁹; similar effects are seen in spin diffusion in Fermi liquids ⁵.

The electrical conductivity is found from σ_{BB} and σ_{WW} by rotating the B and W^3 fields and currents by the Weinberg angle; using Eq. (3), $(J_A, J_{Z^0}) = \mathcal{R}(\theta_W) \sigma \mathcal{R}(-\theta_W)(A, Z^0)$. Thus the electrical conductivity is given by

$$\sigma_{AA} = \sigma_{BB} \cos^2 \theta_W + \sigma_{WW} \sin^2 \theta_W; \quad (5)$$

σ_{el}/T above the electroweak transition differs from that below mainly by a factor $\sim \cos^4 \theta_W \simeq 0.6$.

4 Summary and Outlook

Typically, $\sigma_{el} \simeq T/\alpha \ln(1/\alpha)$, where the logarithmic dependence on the coupling constant arises from Debye and dynamical screening of small momentum-transfer interactions. In the quark-gluon plasma, at $T \gg T_{QGP} \sim 150$ MeV, the additional stopping on quarks reduces the electrical conductivity from that in the hadronic phase. In the electroweak symmetry-restored phase, $T \gg T_c$, interactions between leptons and W^\pm and Z^0 bosons reduce the conductivity further. It does not vanish (as one might have imagined to result from singular unscreened W^\pm -exchanges), and is larger than previous estimates, within an order of magnitude. The current is carried mainly by right-handed leptons since they interact only through exchange of γ and Z^0 .

From the above analysis we can infer the qualitative behavior of other transport coefficients. The characteristic electrical relaxation time, $\tau_{el} \sim (\alpha^2 \ln(1/\alpha) T)^{-1}$, defined from $\sigma_{el} \simeq e^2 n \tau_{el} / T$, is a typical “transport time” which determines relaxation of transport processes when charges are involved. Right-handed leptons interact through Z^0 exchanges only, whereas left-handed leptons may change into neutrinos by W^\pm exchanges as well. Since Z^0 exchange is similar to photon exchange when $T \gg T_c$, the characteristic relaxation time is similar to that for electrical conduction, $\tau_\nu \sim (\alpha^2 \ln(1/\alpha) T)^{-1}$

(except for the dependence on the Weinberg angle). Thus the viscosity is $\eta \sim \tau_\nu \sim T^3/(\alpha^2 \ln(1/\alpha))$. For $T \ll M_W$ the neutrino interaction is suppressed by a factor $(T/M_W)^4$; in this regime neutrinos have longest mean free paths and dominate the viscosity.

The electrical conductivity of the plasma in the early universe is sufficiently large that large-scale magnetic flux present in this period does not diffuse significantly over timescales of the expansion of the universe. The time for magnetic flux to diffuse on a distance scale L is $\tau_{diff} \sim \sigma_{el} L^2$. Since the expansion timescale t_{exp} is $\sim 1/(t_{\text{Planck}} T^2)$, where $t_{\text{Planck}} \sim 10^{-43}$ s is the Planck time, one readily finds that $\frac{\tau_{diff}}{t_{exp}} \sim \alpha x^2 \frac{\tau_{el}}{t_{\text{Planck}}} \gg 1$, where $x = L/ct_{exp}$ is the diffusion length scale in units of the distance to the horizon. As described in Refs. ^{4,14}, sufficiently large domains with magnetic fields in the early universe would survive to produce the primordial magnetic fields observed today.

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References

1. G.A. Baym and H. Heiselberg, *Phys. Rev. D* **56**, 5254 (1997).
2. M.S. Turner and L.M. Widrow, *Phys. Rev.* **D37** (1988) 2743.
3. A. Hosoya and K. Kajantie, *Nucl. Phys.* **B250**, 666 (1985).
4. K. Enqvist, A.I. Rez and V.B. Semikoz, *Nucl. Phys.* **B436** (1995) 49.
5. G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, *Phys. Rev. Lett.* **64**, 1867 (1990); *Nucl. Phys.* **A525**, 415c (1991).
6. G. Baym, H. Heiselberg, C. J. Pethick, and J. Popp, *Nucl. Phys.* **A544**, 569c (1992).
7. H. Heiselberg, *Phys. Rev.* **D49**, 4739 (1994).
8. E. Braaten and M. Thoma, *Phys. Rev.* **D44**, 1298, R2625 (1991).
9. A. Selikhov and M. Gyulassy, *Phys. Lett.* **B316**, 316 (1993). H. Heiselberg, *Phys. Rev. Lett.* **72**, 3013 (1994).
10. C. P. Burgess and A. L. Marini, *Phys. Rev.* **D45**, R17 (1992); A. Rebhan, *Phys. Rev.* **D48**, 482 (1992); R. D. Pisarski, *Phys. Rev.* **D47**, 5589 (1993); H. Heiselberg and C. J. Pethick, *Phys. Rev.* **D47**, R769 (1993).
11. J. P. Blaizot and E. Iancu, *Phys. Rev. Lett.* **76** (1996) 3080; *Phys. Rev.* **D55**, 973 (1997).
12. J. Ahonen and K. Enqvist, *Phys. Lett.* **B382** (1996) 40.
13. K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, *Phys. Rev. Lett.* **77** (1996) 288.
14. G. Baym, D. Bodeker, and L. McLerran, *Phys. Rev.* **D53** (1996) 662.